Crack Length-Effective Stress Intensity Factor Relation in Notched Semi-Circular Specimens for Different Mode of Mixity

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Abstract

The effects of crack inclination angle and crack length on the through-thickness mode I (KI), mode II (KII), and effective (\(K_{eff} = \sqrt{K_{I}^2 + K_{II}^2}\)) stress intensity factors (SIFs) have been analyzed by using three dimensional finite element analysis (3D FEA). Edge crack in semi circular bend specimen (SCB) was utilized in this investigation. The mode of mixity (Me) values are equal to 1 (pure mode I), 0.75, 0.5, 0.25, and 0 (pure mode II). The crack length ratio, crack length/specimen radius (a/R), ranging from 0.1 to 0.7 by step equal to 0.1 has been studied. In SCB specimen, the mode I geometry correction factor (YI) decreased by increasing the crack length for all values of mode of mixity, while, in the case of Me = 1 & 0.75 YI reached its minimum value at a/R = 0.3 then YI increased by increasing the crack length. For all values of Me, Keff increased with the increasing crack length. However, the increment of increment in Keff for a/R ≤ 0.3 is lower than that for a/R > 0.3.

Keywords

Stress Intensity Factor; Mixed Mode I/II; SCB Specimen; Three Dimension Finite Element

Introduction

Disc-type specimens are simple in geometry and have many advantages in terms of specimen preparation, testing and analysis. These test specimens have been used frequently to investigate mixed mode crack growth of rock materials, concrete, biomaterials, and other material[1]. Semi-circular specimens, SCB, (Fig. 1) under three point bending were used for mixed Mode I-II fracture toughness calculation [Khan (2000), Lim (1994), Atkinson (1982)]. Depending on the crack length, its orientation with respect to the loading direction, and the distance between the supports, a variety of mixed-mode failure patterns can be achieved. For pure Mode-I, the crack is aligned parallel to the direction of the load and lies below the loading point. However, for pure Mode-II, a slight misalignment of the specimen and/or crack orientation may induce a component of Mode-I loading and pure Mode-II cannot be achieved [Khan (2000)]. The SCB specimen can give comparable results only when the proper span to diameter ratio is used. The specimen geometry requirement for a valid and representative fracture toughness value or stress intensity factor or other fracture properties of a rock material has been a matter of controversy among researchers. Crack length seems to be a more sensitive factor than specimen thickness on SIFs [Whittaker (1992)]. According to Lim [Lim (1994)], the SIFs become very sensitive at large a/R values for SCB specimens. The effect of specimen thickness and crack length on the variations of mode I and mode II stress intensity factors (SIFs) was analyzed either experimentally or using three dimensional finite element analysis (3D FEA) in Center cracked circular disc specimen (CCCD) and SCB specimen by Sallam et al. [Sallam and Abd-Elhady (2012), Mubarak et al. (2013)]. They found that, the normalized mode I SIF increased at the mid plane of specimen by increasing the thickness of the specimen. However, at the specimen surface, this value decreased by increasing the specimen thickness.

The present study investigated the effects of the sample geometry in terms of crack length and crack inclination angle on mode I and mode II stress intensity factors (SIFs) of edge crack in semi circular bend specimen (SCB), with the aim to make improvements in the testing techniques for fracture testing of brittle materials.

Numerical Analysis

The basic dimensions of SCB specimens R (specimen radius), and B (specimen thickness), were equal to 75 mm and 22.5 respectively. SCB specimen is placed on
two bottom supports of distance 2S and the ratio of S/R was 0.43. The crack length ratio, a/R, ranged from 0.1 to 0.7 by step equal to 0.1. Aliha [Aliha (2010)] suggested five values for mode mixity by parameter called mixity parameter, M:

\[ M_x = \frac{2\pi}{\alpha} \arctan \left( \frac{K_I}{K_{II}} \right) \]  

The values of \( M_x \) varied through 1 (pure mode I), 0.75, 0.5, 0.25, and 0 for pure mode II. Ayatollahi and Aliha [Ayatollahi (2007)] conducted extensive finite element analyses and showed that this mixity parameter was valid by change the crack inclination angle, \( \tau \), as 0, 18.5, 33, 42.5 and 50°.

In the present analysis, the mode I and mode II normalized stress intensity factors are denoted as \( Y_I \) and \( Y_{II} \), respectively, and it can be deduced from Refs. [Hutar (2010), Lim (1993)] that the general formula for normalized stress intensity factor \( Y_i \), is defined as:

\[ Y_i = \frac{4RtK_i}{P\sqrt{a\pi}} \quad i = I, II \]  

Where:

- \( K_I \) = mode I stress intensity factor
- \( K_{II} \) = mode II stress intensity factor
- \( P \) = applied load
- \( a, R, \) and \( t \) = crack length, radius of specimen, and half specimen thickness = \( B/2 \), respectively.

The mechanical properties of the specimen were as follows: modulus of elasticity, \( E = 54 \) Gpa, and poisson's ratio, \( \nu = 0.276 \). The specimen material is homogeneous, isotropic and elastic. The applied load, \( P \) was equal 50 kN as shown in fig. 1, the plane x-y(plane z = 0) is the mid plane of specimen and two specimen surface are \( z = t \) and \( z = -t \), respectively. In order to calculate stress intensity factors of the samples with different geometries, numerical computations were carried out. The package programs used in this study were ABAQUS. ABAQUS program, a finite element program, was used in the present work and all the numerical studies were conducted with 3D models. Disc type specimens require 3D modeling for stress analysis and stress intensity factor computations. For 3D modeling, a choice had to be done among ABAQUS. Around 20 planar layers are divided through the thickness of the SCB specimen. Within each layer, the size of element, C3D8 "8-node linear brick element", decreases gradually with distance from the crack tip decreasing. That is mean FE meshes in the neighborhood of the crack tip are much denser. Figure 2 shows a typical example of the present idealization.

\[ K \]

FIG. 1. GEOMETRY AND LOADING CONDITIONS OF SCB SPECIMENS SUBJECTED TO MIXED MODE I/II LOADING

In the present work, the domain integral method commonly used to extract stress intensity factors (SIFs) [Nakamura (1989, 1991), Gosz (1998, 2002)]. In a finite element, model SIF can be thought of as the virtual motion of a block of material surrounding each node along the crack line. Each such block is defined by contours: each contour is a ring of elements completely surrounding the nodes along the crack line from one crack face to the opposite crack face. These rings of elements are defined recursively to surround all previous contours. ABAQUS/ Standard
automatically finds the elements that form each ring from the regions given as the crack-line definition. Each contour provides an evaluation of the contour integral. Using the divergence theorem, the contour integral can be expanded into a volume integral, over a finite domain surrounding the crack. This domain integral method is used to evaluate contour integrals in ABAQUS/Standard. The method is quite robust in the sense that accurate contour integral estimates are usually obtained even with quite coarse meshes; because the integral is taken over a domain of elements surrounding the crack, errors in local solution parameters have less effect on the evaluated quantities such as the stress intensity factors. The stress intensity factors $K_I$, $K_{II}$ and $K_{III}$ (mode I, mode II and mode III SIF) are usually used in linear elastic fracture mechanics to characterize the local crack-tip/crack-line stress and displacement fields. They are related to the energy release rate (the $J$-integral). The energy release rate is calculated directly in ABAQUS/Standard.

**Results**

Figure 3 shows the results obtained from finite element analysis in the present work for $a/R = 0.3$ and $B/R = 0.1$ and $S/R = 0.43$ and the results given by Ayatollahi and Aliha [Ayatollahi (2007)] at the surface of the SCB specimen, $z = t$. There is a good agreement between the two sets of results and it can be considered as a validation for the present analysis.

![Figure 3. Normalized Mode I and Mode SIFs Calculated in Present Work Compared with Those Presented by Ayatollahi and Aliha (2007) for the SCB Specimen (a/R = 0.3, B/R = 0.1 and S/R = 0.43)](image)

Figure 4 depicts the relation between normalized mode I SIF and crack length at different site at crack front, $z/t$, for SCB specimen with different mode mixity. The value of $Y_i$ increases by decreasing the value of $z/t$, meaning that the value of $Y_i$ at surface point of SCB specimen was not the highest value on the crack front. By increasing the crack length, the value of $Y_i$ decreases to reach a minimum value then it increases that for $M = 0.05$ and 0.25, the value of $Y_i$ decreases by increasing the value of crack length.

![Figure 4. The Effect of SCB Specimen Crack Length on the Normalized Mode I SIF for Different Site on the Crack Front $z/t$ and Different Mode Mixity $M'$ (A) $M' = 1$, (B) $M' = 0.75$, (C) $M' = 0.5$ and (D) $M' = 0.25$.](image)

![Figure 5. The Effect of SCB Specimen Crack Length on the Normalized Mode II SIF for Different Site on the Crack Front $z/t$ and Different Mode Mixity $M'$ (A) $M' = 1$, (B) $M' = 0.75$, (C) $M' = 0.5$ and (D) $M' = 0.25$.](image)
A little effect of $z/t$ on the value of normalized mode II SIF, $Y_{II}$, with different crack length can be shown in Fig. 5. The value of $Y_{II}$ decreases by increasing the value of crack length to reach the minimum value then increases by increasing the crack length that for $M_r = 0.5, 0.25$ and $0$. However, for $M_r = 0.75$ the value of $Y_{II}$ increases by increasing the crack length.

Figure 6 shows the effect of crack length, $a/R$ on the effective stress intensity factor, $K_{eff}$, ($K_{eff} = \sqrt{K^2_I + K^2_{II}}$) through the SCB specimen crack front. The change of the effective stress intensity factor, $K_{eff}$, ($K_{eff} = \sqrt{K^2_I + K^2_{II}}$) through the SCB specimen crack front can be neglected. By increasing the crack length, the values of $K_{eff}$ increase. In the case of pure mode I and II, the effective stress intensity factor rises by increasing the crack length.

**Conclusions**

The present numerical analysis revealed the following conclusions:

1- For $a/R < 0.3$, the mode I geometry correction factor ($Y_I$) decreased by increasing the crack length for all values of mode of mixity.

2- For $a/R > 0.3$, $Y_{II}$ increased by increasing the crack length for $M_r = 1$ and $0.75$, while, $Y_{II}$ continued to decrease with the increasing crack length for $M_r = 0.5$ and $0.25$.

3- For $a/R > 0.3$, $Y_{II}$ increased by increasing the crack length for all values of $Me$.

4- For all values of $M_r$, $K_{eff}$ increased with the increasing crack length. However, the increment of increment in $K_{eff}$ for $a/R \leq 0.3$ was lower than that for $a/R > 0.3$.

**REFERENCES**


